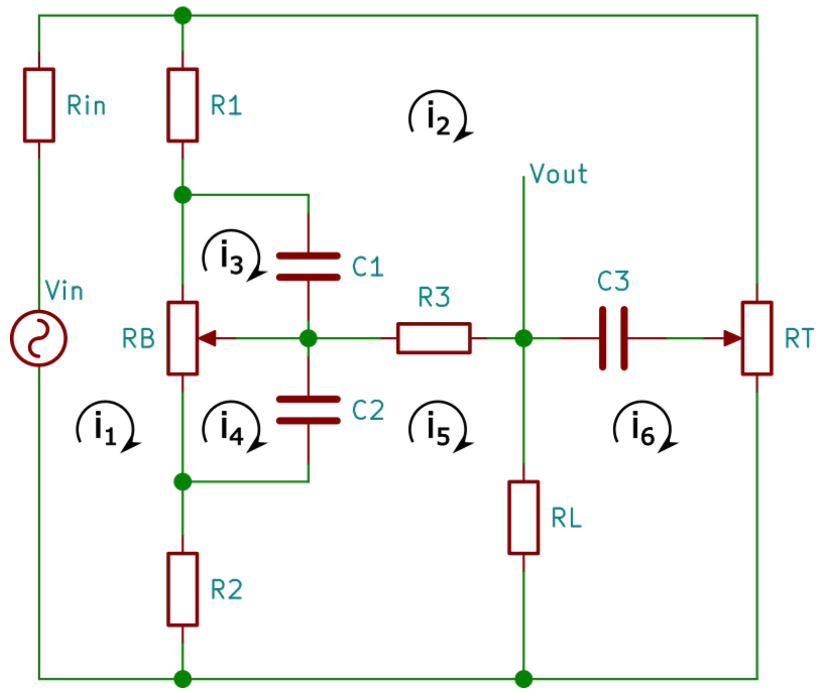
Circuit Analysis of Passive Baxandall

To find the frequency response of the circuit, the ratio $\frac{V_{out}}{V_{in}}$ needs to be determined. Mesh analysis is performed to yield a system of linear equations, which are then placed in matrix form and solved using Cramer's rule.



Mesh Analysis

Loop #1: Bass potentiometer R_B is modeled as two resistors, R_{B1} and R_{B2}, connected at the wiper. Using Kirchhoff's voltage law (KVL),

$$i_1(R_{in} + R_1 + R_{B1} + R_{B2} + R_2) + i_2(-R_1) + i_3(-R_{B1}) + i_4(-R_{B2}) + i_5(-R_2) = V_{in}$$

Loop #2: Treble potentiometer R_T is modeled as two resistors, R_{T1} and R_{T2}, connected at the wiper.

$$i_1(-R_1) + i_2(R_1 + R_3 + R_{T1} + \frac{1}{j\omega C_1} + \frac{1}{j\omega C_3}) + i_3(-\frac{1}{j\omega C_1}) + i_5(-R_3) + i_6(-\frac{1}{j\omega C_3}) = 0$$

Loop #3: $i_1(-R_{B1}) + i_2(-\frac{1}{j\omega C_1}) + i_3(R_{B1} + \frac{1}{j\omega C_1}) = 0$

Loop #4: $i_1(-R_{B2}) + i_4(R_{B2} + \frac{1}{j\omega C_2}) + i_5(-\frac{1}{j\omega C_2}) = 0$

Loop #5: $i_1(-R_2) + i_2(-R_3) + i_4(-\frac{1}{j\omega C_2}) + i_5(R_2 + R_3 + R_L + \frac{1}{j\omega C_2}) + i_6(-R_L) = 0$

Loop #6: $i_2(-\frac{1}{j\omega C_3}) + i_5(-R_L) + i_6(R_{T2} + R_L + \frac{1}{j\omega C_3}) = 0$

Matrix Form

There are now 6 node equations with 6 current variables. These can be restated in matrix form

$$Ax = b$$

where A is a 6 x 6 matrix of the coefficients (impedances), x is a column vector of the variables (loop currents), and b is a column vector of the right-hand sides of the equations (inputs and constants).

$$\begin{bmatrix} R_{in}+R_1+R_{B1}+R_{B2}+R_2 & -R_1 & -R_{B1} & -R_{B2} & -R_2 & 0 \\ -R_1 & R_1+R_3+R_{T1}+\frac{1}{j\omega C_1}+\frac{1}{j\omega C_3} & -\frac{1}{j\omega C_1} & 0 & -R_3 & -\frac{1}{j\omega C_3} \\ -R_{B1} & -\frac{1}{j\omega C_1} & R_{B1}+\frac{1}{j\omega C_1} & 0 & 0 & 0 \\ -R_{B2} & 0 & 0 & R_{B2}+\frac{1}{j\omega C_2} & -\frac{1}{j\omega C_2} & 0 \\ -R_2 & -R_3 & 0 & -\frac{1}{j\omega C_2} & R_2+R_3+R_L+\frac{1}{j\omega C_2} & -R_L \\ 0 & -\frac{1}{j\omega C_3} & 0 & 0 & -R_L & R_{T2}+R_L+\frac{1}{j\omega C_3} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{bmatrix} = \begin{bmatrix} V_{in} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The output voltage, V_{out}, is the voltage drop across load resistor R_L, meaning $V_{out} = (i_5 - i_6)R_L$. Currents i_5 and i_6 can be found using Cramer's rule.

$$i_5 = \frac{|A_5|}{|A|} \qquad i_6 = \frac{|A_6|}{|A|}$$

where A_5 and A_6 are matrices formed by replacing the 5th and 6th columns of A, respectively, with the contents of b. Since b contains only one non-zero element, the determinants of A_5 and A_6 are equal to that element multiplied by the cofactors.

$$|A_5| = V_{in}C_{1,5} = V_{in}(-1)^{1+5}(M_{1,5})$$
$$|A_6| = V_{in}C_{1,6} = V_{in}(-1)^{1+6}(M_{1,6})$$

where $M_{1,5}$ is the determinant of A_5 with row 1 and column 5 removed, and $M_{1,6}$ is the determinant of A_6 with row 1 and column 6 removed.

$$M_{1,5} = \begin{vmatrix} -R_1 & R_1+R_3+R_{T1}+\frac{1}{j\omega C_1}+\frac{1}{j\omega C_3} & -\frac{1}{j\omega C_1} & 0 & -\frac{1}{j\omega C_3} \\ -R_{B1} & -\frac{1}{j\omega C_1} & R_{B1}+\frac{1}{j\omega C_1} & 0 & 0 \\ -R_{B2} & 0 & 0 & R_{B2}+\frac{1}{j\omega C_2} & 0 \\ -R_2 & -R_3 & 0 & -\frac{1}{j\omega C_2} & -R_L \\ 0 & -\frac{1}{j\omega C_3} & 0 & 0 & R_{T2}+R_L+\frac{1}{j\omega C_3} \end{vmatrix}$$

$$M_{1,6} = \begin{vmatrix} -R_1 & R_1+R_3+R_{T1}+\frac{1}{j\omega C_1}+\frac{1}{j\omega C_3} & -\frac{1}{j\omega C_1} & 0 & -R_3 \\ -R_{B1} & -\frac{1}{j\omega C_1} & R_{B1}+\frac{1}{j\omega C_1} & 0 & 0 \\ -R_{B2} & 0 & 0 & R_{B2}+\frac{1}{j\omega C_2} & -\frac{1}{j\omega C_2} \\ -R_2 & -R_3 & 0 & -\frac{1}{j\omega C_2} & R_2+R_3+R_L+\frac{1}{j\omega C_2} \\ 0 & -\frac{1}{j\omega C_3} & 0 & 0 & -R_L \end{vmatrix}$$

Substituting into the equation for V_{out},

$$V_{out} = (i_5 - i_6)R_L = R_L\frac{|A_5|-|A_6|}{|A|} = R_L V_{in}\frac{M_{1,5}+M_{1,6}}{|A|}.$$

The transfer function of the circuit is then found to be

$$\frac{V_{out}}{V_{in}} = R_L\frac{M_{1,5}+M_{1,6}}{|A|}.$$