## Analysis of Neve High/Low Shelving Tone Control Circuit

As used in the 1066, 1073, 1084 and more

The Neve high/low shelving circuit is a variation on a Baxandall tone control. The vintage design features a custom, discrete inverting amplifier. For ease of analysis, an ideal opamp can be substituted as an approximation.

To find the frequency response of the circuit, the ratio $\frac{V_{\text {out }}}{V_{\text {in }}}$ needs to be determined. Nodal analysis is performed to yield a system of linear equations, which are then placed in matrix form and solved using Cramer's rule.


Note that input resistor $R_{i n}$ is not part of the tone control circuit proper, but represents the output impedance of the circuit providing the input signal. To get a flat response with the potentiometers set halfway, feedback resistor $R_{F}$ can be set to match $R_{i n}$.

## Nodal Analysis

Treble potentiometer $R_{T}$ is modeled as resistors $R_{T 1}$ and $R_{T 2}$, connected at the wiper. Since $R_{4}, C_{4}$, and $R_{T 1}$ are in series between nodes 1 and 5 , their impedances may be combined as $Z_{T 1}$. Likewise, $R_{5}, C_{5}$, and $R_{T 2}$ may be represented as $Z_{T 2}$. Using Laplace variable $s$ (complex frequency):

$$
\begin{aligned}
Z_{T 1} & =R_{4}+R_{T 1}+\frac{1}{s C_{4}} \\
Z_{T 2} & =R_{5}+R_{T 2}+\frac{1}{s C_{5}}
\end{aligned}
$$

Node \#1: Using Kirchhoff's current law (KCL),

$$
v_{1}\left(\frac{1}{R_{i n}}+\frac{1}{R_{1}}+\frac{1}{Z_{T 1}}\right)+v_{2}\left(-\frac{1}{R_{1}}\right)+v_{5}\left(-\frac{1}{Z_{T 1}}\right)=\frac{V_{i n}}{R_{i n}} .
$$

Node \#2: Bass potentiometer $R_{B}$ is modeled as two resistors, $R_{B 1}$ and $R_{B 2}$, connected at the wiper.

$$
v_{1}\left(-\frac{1}{R_{1}}\right)+v_{2}\left(\frac{1}{R_{1}}+\frac{1}{R_{B 1}}+s C_{1}\right)+v_{3}\left(-\frac{1}{R_{B 1}}\right)+v_{7}\left(-s C_{1}\right)=0
$$

Node \#3: The positive and negative inputs of an ideal opamp are modeled as having no voltage difference between them. Since the positive input is grounded, the negative input (and therefore the connection to $R_{3}$ at node 8 ) is considered grounded as well.

$$
v_{2}\left(-\frac{1}{R_{B 1}}\right)+v_{3}\left(\frac{1}{R_{3}}+\frac{1}{R_{B 1}}+\frac{1}{R_{B 2}}+s C_{6}\right)+v_{4}\left(-\frac{1}{R_{B 2}}\right)+v_{7}\left(-s C_{6}\right)=0
$$

## Node \#4:

$$
v_{3}\left(-\frac{1}{R_{B 2}}\right)+v_{4}\left(\frac{1}{R_{2}}+\frac{1}{R_{B 2}}+s C_{2}\right)+v_{6}\left(-\frac{1}{R_{2}}\right)+v_{7}\left(-s C_{2}\right)=0
$$

Node \#5: As with $R_{3}$ above, the connection of $C_{3}$ at node 8 is considered grounded since it is connected to the negative opamp input.

$$
v_{1}\left(-\frac{1}{Z_{T 1}}\right)+v_{5}\left(\frac{1}{Z_{T 1}}+\frac{1}{Z_{T 2}}+s C_{3}\right)+v_{6}\left(-\frac{1}{Z_{T 2}}\right)=0
$$

Node \#6:

$$
v_{4}\left(-\frac{1}{R_{2}}\right)+v_{5}\left(-\frac{1}{Z_{T 2}}\right)+v_{6}\left(\frac{1}{R_{2}}+\frac{1}{Z_{T 2}}+\frac{1}{R_{F}}\right)+V_{\text {out }}\left(-\frac{1}{R_{F}}\right)=0
$$

Node \#7:

$$
v_{2}\left(-s C_{1}\right)+v_{3}\left(-s C_{6}\right)+v_{4}\left(-s C_{2}\right)+v_{7}\left(\frac{1}{R_{6}}+s C_{1}+s C_{2}+s C_{6}\right)=0
$$

Node $\# 8$ : The inputs of an ideal opamp are modeled as conducting no current.

$$
v_{3}\left(-\frac{1}{R_{3}}\right)+v_{5}\left(-s C_{3}\right)=0
$$

## Matrix Form

There are 8 node equations with 8 node voltage variables. These can be stated in matrix form

$$
A x=b
$$

where $A$ is a matrix of the coefficients (admittances), $x$ is a column vector of the variables (node voltages), and $b$ is a column vector of the right-hand sides of the equations (inputs and constants).

$$
\left[\begin{array}{cccccccc}
Y_{1} & -\frac{1}{R_{1}} & 0 & 0 & -\frac{1}{Z_{T 1}} & 0 & 0 & 0 \\
-\frac{1}{R_{1}} & Y_{2} & -\frac{1}{R_{B 1}} & 0 & 0 & 0 & -s C_{1} & 0 \\
0 & -\frac{1}{R_{B 1}} & Y_{3} & -\frac{1}{R_{B 2}} & 0 & 0 & -s C_{6} & 0 \\
0 & 0 & -\frac{1}{R_{B 2}} & Y_{4} & 0 & -\frac{1}{R_{2}} & -s C_{2} & 0 \\
-\frac{1}{Z_{T 1}} & 0 & 0 & 0 & Y_{5} & -\frac{1}{Z_{T 2}} & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{R_{2}} & -\frac{1}{Z_{T 2}} & Y_{6} & 0 & -\frac{1}{R_{F}} \\
0 & -s C_{1} & -s C_{6} & -s C_{2} & 0 & 0 & Y_{7} & 0 \\
0 & 0 & -\frac{1}{R_{3}} & 0 & -s C_{3} & 0 & 0 & Y_{8}
\end{array}\right]\left[\begin{array}{c}
v_{1} \\
v_{2} \\
v_{3} \\
v_{4} \\
v_{5} \\
v_{6} \\
v_{7} \\
V_{\text {out }}
\end{array}\right]=\left[\begin{array}{c}
\frac{V_{i n}}{R_{i n}} \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{array}\right]
$$

For readability, the admittances along the diagonal of $A$ are given separately here.

$$
\begin{array}{ll}
Y_{1}=\frac{1}{R_{i n}}+\frac{1}{R_{1}}+\frac{1}{Z_{T 1}} & Y_{5}=\frac{1}{Z_{T 1}}+\frac{1}{Z_{T 2}}+s C_{3} \\
Y_{2}=\frac{1}{R_{1}}+\frac{1}{R_{B 1}}+s C_{1} & Y_{6}=\frac{1}{R_{2}}+\frac{1}{Z_{T 2}}+\frac{1}{R_{F}} \\
Y_{3}=\frac{1}{R_{3}}+\frac{1}{R_{B 1}}+\frac{1}{R_{B 2}}+s C_{6} & Y_{7}=\frac{1}{R_{6}}+s C_{1}+s C_{2}+s C_{6} \\
Y_{4}=\frac{1}{R_{2}}+\frac{1}{R_{B 2}}+s C_{2} & Y_{8}=-s C_{F}
\end{array}
$$

The output voltage, $V_{\text {out }}$, can now be found using Cramer's rule.

$$
V_{\text {out }}=\frac{\left|A_{8}\right|}{|A|}
$$

where matrix $A_{8}$ is formed by replacing the 8 th column of $A$ with the contents of $b$. Since $b$ contains only one non-zero element, the determinant of $A_{8}$ is equal to that element multiplied by its cofactor.

$$
\begin{aligned}
& \left|A_{8}\right|=\frac{V_{i n}}{R_{i n}} C_{1,8}=\frac{V_{i n}}{R_{i n}}(-1)^{1+8}\left(M_{1,8}\right) \\
& \left|A_{8}\right|=-\frac{V_{i n}}{R_{i n}} M_{1,8}
\end{aligned}
$$

where $M_{1,8}$ is the determinant of $A_{8}$ with row 1 and column 8 removed.

$$
M_{1,8}=\left|\begin{array}{ccccccc}
-\frac{1}{R_{1}} & Y_{2} & -\frac{1}{R_{B 1}} & 0 & 0 & 0 & -s C_{1} \\
0 & -\frac{1}{R_{B 1}} & Y_{3} & -\frac{1}{R_{B 2}} & 0 & 0 & -s C_{6} \\
0 & 0 & -\frac{1}{R_{B 2}} & Y_{4} & 0 & -\frac{1}{R_{2}} & -s C_{2} \\
-\frac{1}{Z_{T 1}} & 0 & 0 & 0 & Y_{5} & -\frac{1}{Z_{T 2}} & 0 \\
0 & 0 & 0 & -\frac{1}{R_{2}} & -\frac{1}{Z_{T 2}} & Y_{6} & 0 \\
0 & -s C_{1} & -s C_{6} & -s C_{2} & 0 & 0 & Y_{7} \\
0 & 0 & -\frac{1}{R_{3}} & 0 & -s C_{3} & 0 & 0
\end{array}\right|
$$

Substituting into the equation for $V_{\text {out }}$,

$$
V_{\text {out }}=-\frac{V_{\text {in }}}{R_{\text {in }}} \frac{M_{1,8}}{|A|}
$$

The transfer function of the circuit is then found to be

$$
\frac{V_{\text {out }}}{V_{\text {in }}}=-\frac{1}{R_{\text {in }}} \frac{M_{1,8}}{|A|} .
$$

