A systematic method for calculating transistor circuit biasing voltages

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In this context a few practical examples are given to demonstrate a systematic approach to solve all biasing voltages and currents from a bipolar transistor circuit of any degree of complexity. The approach uses Kirchhoff's current law as basis for constructing voltage equations for each node of the circuit under analysis. In the following sections, four practical examples are given to explain the proposed method. There of course are other approaches as well, but this is the most systematic one to my knowledge.

Often the bias analysis can be done directly from the schematic, but sometimes it is necessary to replace the transistor with its DC equivalent circuit and construct a current equation for the emitter voltage node V_E. Typically one can use the base or emitter voltage as a so-called supernode, which combines the two nodes with the relation $V_B = V_E + V_{BE}$, where $V_{BE} \approx 0.6$ V is the typical junction voltage of a pn-junction when the transistor is biased in forward-active region. The following examples should indicate when it is necessary to use the equivalent model for the transistor.

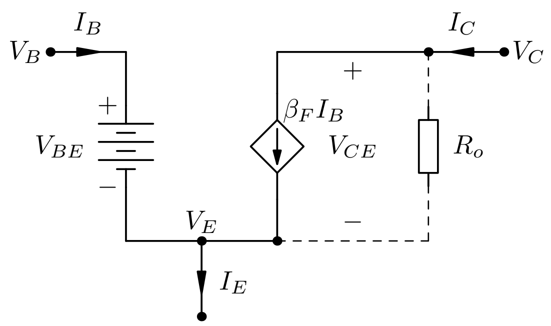

Figure 1: A DC equivalent model of a bipolar junction transistor

1 A basic common-emitter BJT amplifier

The first simple example concerns the basic emitter-coupled amplifier in Figure 2. This schematic includes many components that do not take part in the DC biasing of the transistor. Therefore, it is necessary to prepare a separate drawing, which only includes the DC part of the circuit.

Figure 3 indicates the components that affect the static currents and voltages in this BJT amplifier. To make the analysis easier, it is better to include clear namings for all the currents and voltage nodes of the circuit. When replacing the transistor with its equivalent model, one notices that the circuit in Figure 3 actually has two voltage nodes, V_B and V_E.

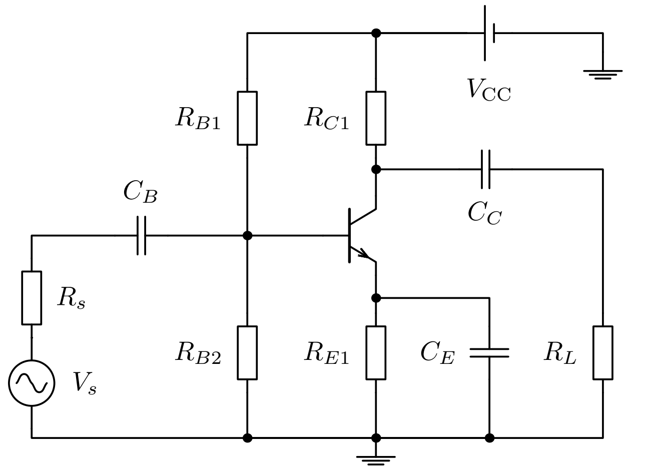

Figure 2: A basic common-emitter BJT amplifier

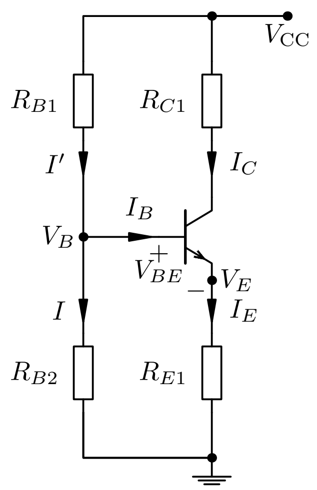

Figure 3: A DC model of the basic BJT amplifier

However, since in this case V_B and V_E are related by the pn-junction voltage V_{BE}, the circuit has only one distinct voltage node V_B. This means that only one current equation needs to be written for that node and all the bias voltages of the circuit can be solved using that single voltage value. It is important that there are as many independent current equations as there are voltage nodes in the circuit.

According to the Kirchhoff's current rule, the current equation for the voltage node V_B is:

$$I' = I + I_B = I + \frac{I_E}{\beta_F + 1} \tag{1}$$

After the current equation is written down, the next step is to express the currents using the supply voltage and the node voltages. The currents appearing in equation (1) can be expressed with respect to the node voltage as:

$$I_E = \frac{V_B - V_{BE}}{R_{E1}} \quad ; \quad I' = \frac{V_{CC} - V_B}{R_{B1}} \quad ; \quad I = \frac{V_B}{R_{B2}}$$

and after substituting these voltage equations to the current equation,

$$\frac{V_{CC} - V_B}{R_{B1}} = \frac{V_B}{R_{B2}} + \frac{V_B - V_{BE}}{R_{E1}(\beta_F + 1)}.$$

From this equation it is possible to solve the node voltage V_B. With the fact that $V_E = V_B - V_{BE}$, one has an equation from where other biasing voltages and currents can be solved.

2 A direct-coupled pair with current-shunt feedback

The circuit shown in Figure 4 is used as a basis for a few guitar effect circuits, including the famous 'Fuzz-Face' effect pedal.

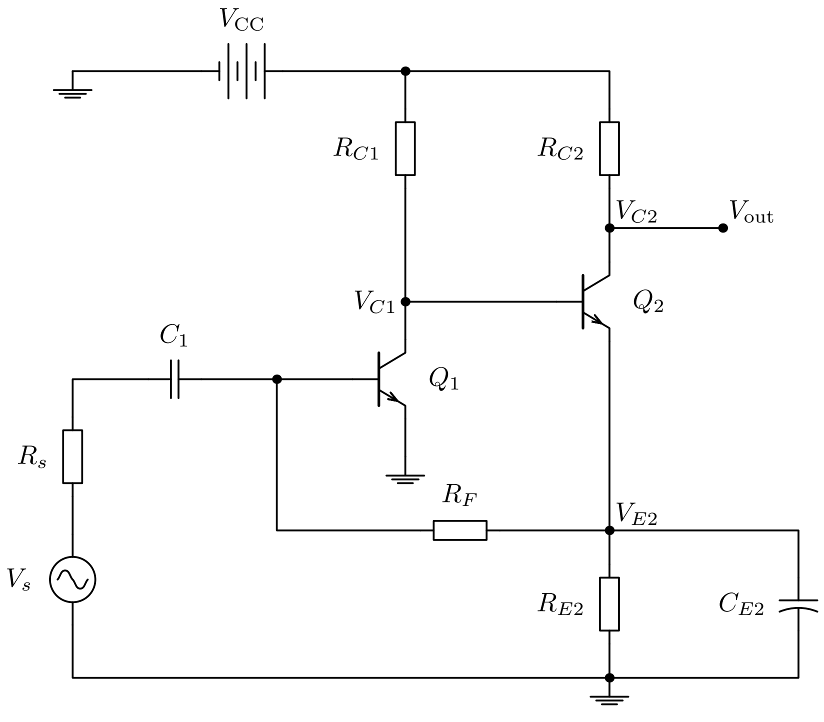

Figure 4: A direct-coupled pair amplifier with current-shunt feedback

The DC part of this circuit is shown in Figure 5. The direct current model in Figure 5 has two distinct voltage nodes, labelled as V_{C1} and V_{E2}. The output node is identified in this case as V_{C2}, but it is not needed in the bias analysis. It is also beneficial to notice that $V_{E2} = V_{C1} - V_{BE2}$. This reduces the number of distinct voltage nodes to one.

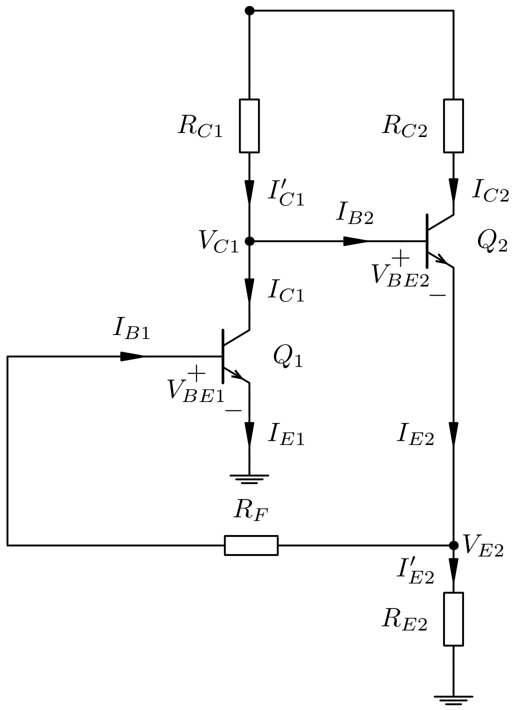

Figure 5: A DC model of the amplifier with current-shunt feedback

Based on Kirchhoff's current law, the sum of currents entering a node equals the sum of currents leaving a node. Therefore, the current equations for the voltage nodes V_{E2} and V_{C1} are:

$$I'_{E2} = I_{E2} - I_{B1} = (\beta_{F2} + 1)I_{B2} - I_{B1}$$
$$I'_{C1} = I_{C1} + I_{B2} = \beta_{F1} I_{B1} + I_{B2}.$$

These equations can be combined to eliminate the base current I_{B2} and then the current equation reads

$$I'_{C1} = \beta_{F1} I_{B1} + \frac{I'_{E2}}{\beta_{F2} + 1} + \frac{I_{B1}}{\beta_{F2} + 1}.$$

These currents can be expressed with the node voltages as:

$$I'_{E2} = \frac{V_{E2}}{R_{E2}} \quad ; \quad I'_{C1} = \frac{V_{\mathrm{CC}} - V_{C1}}{R_{C1}} \quad ; \quad I_{B1} = \frac{V_{E2} - V_{BE1}}{R_F}$$

and after substituting the voltage equations,

$$\frac{V_{\mathrm{CC}} - V_{C1}}{R_{C1}} = \beta_{F1} \frac{V_{E2} - V_{BE1}}{R_F} + \frac{V_{E2}}{R_{E2}(\beta_{F2} + 1)} + \frac{V_{E2} - V_{BE1}}{R_F(\beta_{F2} + 1)}.$$

With the fact that $V_{E2} = V_{C1} - V_{BE2}$, one has an equation from where the voltage V_{C1} can be solved. Based on this analysis, all the other biasing voltages and currents can also be solved.

3 A direct-coupled pair with voltage-series feedback

The circuit shown in Figure 6 is basically same as the current-shunt circuit in the previous section, but now the feedback loop is connected to implement voltage-series feedback.

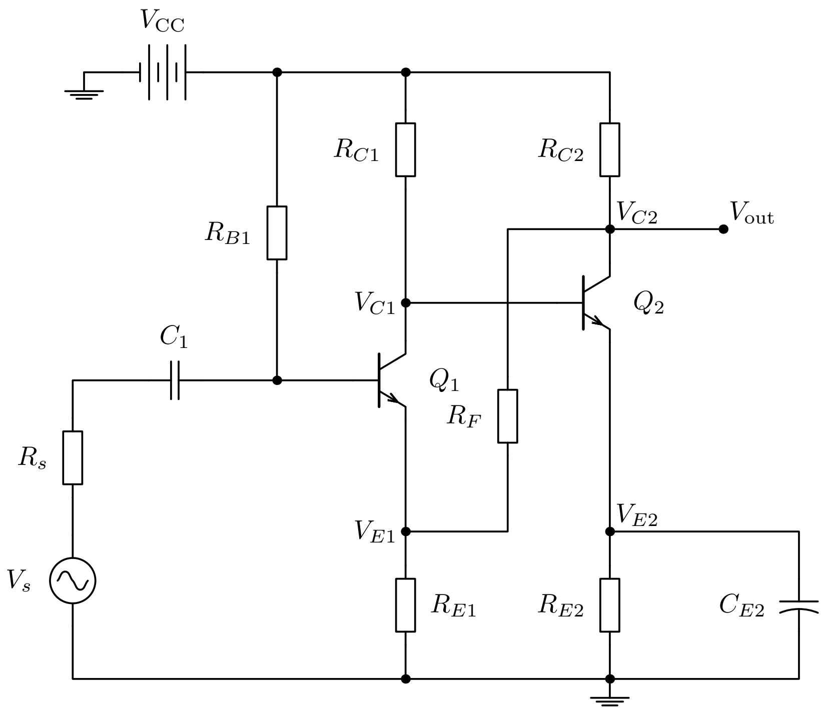

Figure 6: A direct-coupled pair amplifier with voltage-series feedback

The DC model is redrawn for analysis purposes in Figure 7. The analysis assumes that both of the transistors are biased in the forward-active region. The biasing is however sensitive to the current levels and either of the transistors is easily saturated in this connection. In this case, the feedback loop affects very little to the DC biasing, because the feedback current only affects the potential V_{E1} and does not interfere with any significant transistor bias current.

The circuit in Figure 5 has three distinct voltage nodes, labelled as V_{C1}, V_{E1} and V_{C2}. Based on Kirchhoff's current law, the current equations for the nodes are:

$$I'_{E1} = I_{E1} + I_F = (\beta_{F1} + 1)I_{B1} + I_F$$
$$I'_{C1} = I_{C1} + I_{B2} = \beta_{F1} I_{B1} + I_{B2}$$
$$I'_{C2} = I_{C2} + I_F = \beta_{F2} I_{B2} + I_F$$

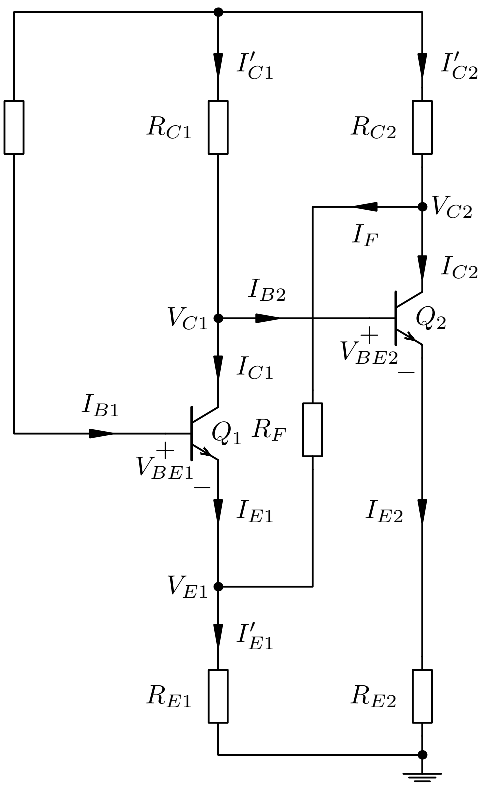

Figure 7: A DC model of the amplifier with voltage-series feedback

The essential currents can be expressed with the node voltages as:

$$I'_{E1} = \frac{V_{E1}}{R_{E1}} \quad ; \quad I'_{C1} = \frac{V_{CC} - V_{C1}}{R_{C1}} \quad ; \quad I_{B1} = \frac{V_{CC} - V_{BE1} - V_{E1}}{R_{B1}}$$

$$I_F = \frac{V_{C2} - V_{E1}}{R_F} \quad ; \quad I'_{C2} = \frac{V_{CC} - V_{C2}}{R_{C2}} \quad ; \quad I_{B2} = \frac{V_{C1} - V_{BE2}}{R_{E2}(\beta_{F2}+1)}.$$

Substitutions of the voltage equations into the current equations leads to the following three equations

$$\frac{V_{E1}}{R_{E1}} = (\beta_{F1}+1)\frac{V_{CC} - V_{BE1} - V_{E1}}{R_{B1}} + \frac{V_{C2} - V_{E1}}{R_F}$$

$$\frac{V_{CC} - V_{C1}}{R_{C1}} = \beta_{F1}\frac{V_{CC} - V_{BE1} - V_{E1}}{R_{B1}} + \frac{V_{C1} - V_{BE2}}{R_{E2}(\beta_{F2}+1)}$$

$$\frac{V_{CC} - V_{C2}}{R_{C2}} = \frac{\beta_{F2}}{\beta_{F2}+1}\frac{V_{C1} - V_{BE2}}{R_{E2}(\beta_{F2}+1)} + \frac{V_{C2} - V_{E1}}{R_F}.$$

These equations can be organised into a matrix equation, from where each of the node voltages can be solved using the Cramer's rule. The matrix equation is of the form:

$$\begin{bmatrix} \frac{1}{R_{E1}} + \frac{\beta_{F1}+1}{R_{B1}} + \frac{1}{R_F} & 0 & -\frac{1}{R_F} \\ -\frac{\beta_{F1}}{R_{B1}} & \frac{1}{R_{C1}} + \frac{1}{R_{E2}(\beta_{F2}+1)} & 0 \\ -\frac{1}{R_F} & \frac{\beta_{F2}}{\beta_{F2}+1}\frac{1}{R_{E2}} & \frac{1}{R_{C2}} + \frac{1}{R_F} \end{bmatrix} \times \begin{bmatrix} V_{E1} \\ V_{C1} \\ V_{C2} \end{bmatrix} = \begin{bmatrix} (\beta_{F1}+1)\frac{V_{CC}-V_{BE1}}{R_{B1}} \\ \frac{V_{CC}}{R_{C1}} + \beta_{F1}\frac{V_{BE1}-V_{CC}}{R_{B1}} + \frac{V_{BE2}}{R_{E2}(\beta_{F2}+1)} \\ \frac{V_{CC}}{R_{C1}} + \frac{\beta_{F2}}{\beta_{F2}+1}\frac{V_{BE2}}{R_{E2}} \end{bmatrix}.$$

Based on this analysis, all the other biasing voltages and currents can be solved using the magnitudes of the node voltages.

4 The Vox-type wah circuit

The circuit in Figure 8 is also a variation of the current-shunt feedback amplifier, but this version has a more complex biasing network.

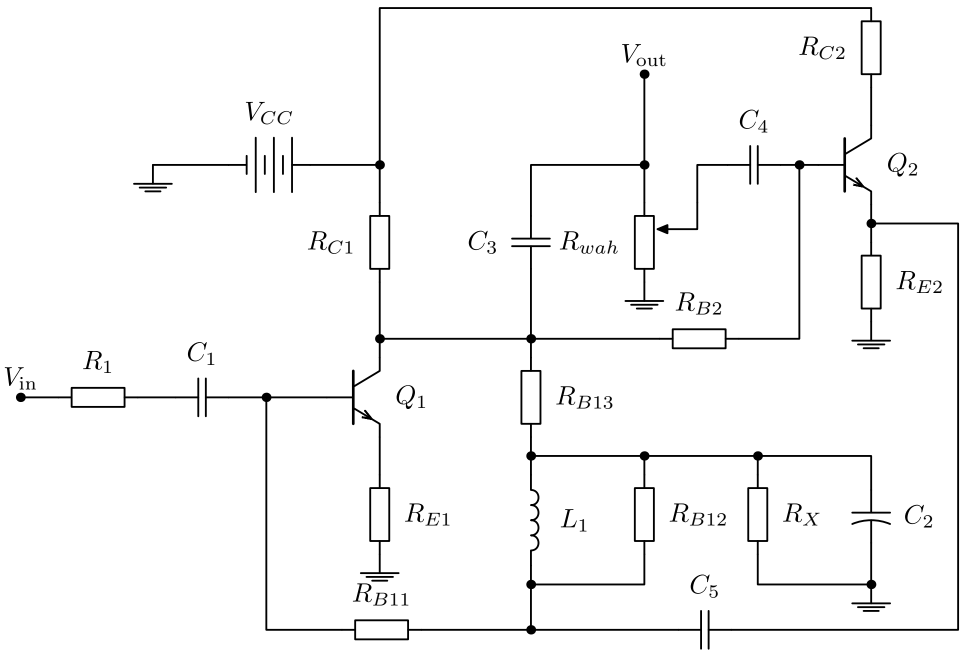

Figure 8: The Vox wah guitar effect circuit diagram

The DC part of the circuit 8 is redrawn in Figure 9 for analysis purposes. The analysis assumes that both of the transistors are biased in the forward-active region. The obvious voltage nodes of the circuit are V_{C1} and V'_{B1} but those will not give enough information about all the branches of the circuit. In this case it is better to utilise the DC equivalent model of the BJT and add both emitter voltage nodes V_{E1} and V_{E2} to the set. Now there should be enough information to solve the biasing voltages of the circuit.

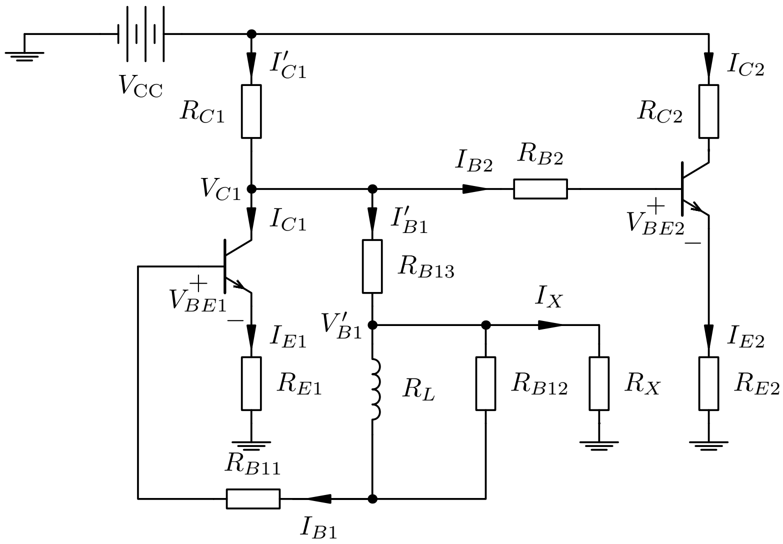

Figure 9: The DC section of the wah circuit

Because the internal resistance R_L of the inductor is much smaller than the resistance of R_{B12} the parallel resistance equals R_L. This small resistance could be completely neglected in the following analysis, but in this case it is kept in the calculations.

Based on Kirchhoff's current law, the sum of currents entering a node equals the sum of currents leaving a node. Therefore, the current equations for the voltage nodes V_{C1}, V'_{B1}, V_{E1} and V_{E2} are:

$$I'_{C1} = I_{C1} + I_{B2} + I'_{B1}$$
$$I'_{B1} = I_{B1} + I_X$$
$$I_{E1} = (\beta_{F1} + 1)I_{B1}$$
$$I_{E2} = (\beta_{F2} + 1)I_{B2}$$

The essential currents in the above equations can be expressed with the node voltages as:

$$I'_{B1} = \frac{V_{C1} - V_{B1}}{R_{B13}} \quad ; \quad I'_{C1} = \frac{V_{CC} - V_{C1}}{R_{C1}} \quad ; \quad I_{B1} = \frac{V'_{B1} - V_{BE1} - V_{E1}}{R_{B11} + R_L}$$
$$I_X = \frac{V'_{B1}}{R_X} \quad ; \quad I_{B2} = \frac{V_{C1} - V_{BE2} - V_{E2}}{R_{B2}} \quad ; \quad I_{E1} = \frac{V_{E1}}{R_{E1}}$$

and I_{E2} is similar to I_{E1}. Substitutions of the voltage equations into the current equations leads to the following four equations

$$\frac{V_{\mathrm{CC}} - V_{C1}}{R_{C1}} = \frac{\beta_{F1}}{\beta_{F1}+1}\frac{V_{E1}}{R_{E1}} + \frac{V_{C1} - V_{BE2} - V_{E2}}{R_{B2}} + \frac{V_{C1} - V'_{B1}}{R_{B13}}$$

$$\frac{V_{C1} - V'_{B1}}{R_{B13}} = \frac{V'_{B1} - V_{BE1} - V_{E1}}{R_{B11} + R_L} + \frac{V'_{B1}}{R_X}$$

$$\frac{V_{E1}}{R_{E1}} = (\beta_{F1}+1)\frac{V'_{B1} - V_{BE1} - V_{E1}}{R_{B11} + R_L}$$

$$\frac{V_{E2}}{R_{E2}} = (\beta_{F2}+1)\frac{V_{C1} - V_{BE2} - V_{E2}}{R_{B2}}.$$

These equations can be organised into a matrix equation, from where each of the node voltages can be solved using the Cramer's rule. The matrix equation is of the form:

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & 0 \\ 0 & Y_{32} & Y_{33} & 0 \\ Y_{41} & 0 & 0 & Y_{44} \end{bmatrix} \times \begin{bmatrix} V_{C1} \\ V'_{B1} \\ V_{E1} \\ V_{E2} \end{bmatrix} = \begin{bmatrix} \frac{V_{\mathrm{CC}}}{R_{C1}} + \frac{V_{BE2}}{R_{B2}} \\ \frac{V_{BE1}}{R_{B11}+R_L} \\ \frac{V_{BE1}}{R_{B11}+R_L} \\ \frac{V_{BE2}}{R_{B2}} \end{bmatrix}$$

where the non-zero terms in the admittance matrix are

$$Y_{11} = \frac{1}{R_{C1}} + \frac{1}{R_{B2}} + \frac{1}{R_{B13}} \qquad Y_{12} = Y_{21} = -\frac{1}{R_{B13}}$$

$$Y_{13} = \frac{\beta_{F1}}{\beta_{F1}+1}\frac{1}{R_{E1}} \qquad Y_{14} = -\frac{1}{R_{B2}}$$

$$Y_{22} = \frac{1}{R_{B13}} + \frac{1}{R_{B11}+R_L} + \frac{1}{R_X} \qquad Y_{23} = -\frac{1}{R_{B11}+R_L}$$

$$Y_{32} = \frac{1}{R_{B11}+R_L} \qquad Y_{33} = -\frac{1}{R_{B11}+R_L} - \frac{1}{R_{E1}(\beta_{F1}+1)}$$

$$Y_{41} = \frac{1}{R_{B2}} \qquad Y_{44} = -\frac{1}{R_{B2}} - \frac{1}{R_{E2}(\beta_{F2}+1)}$$

Based on this analysis, all the other biasing voltages and currents can be solved using the magnitudes of the node voltages.