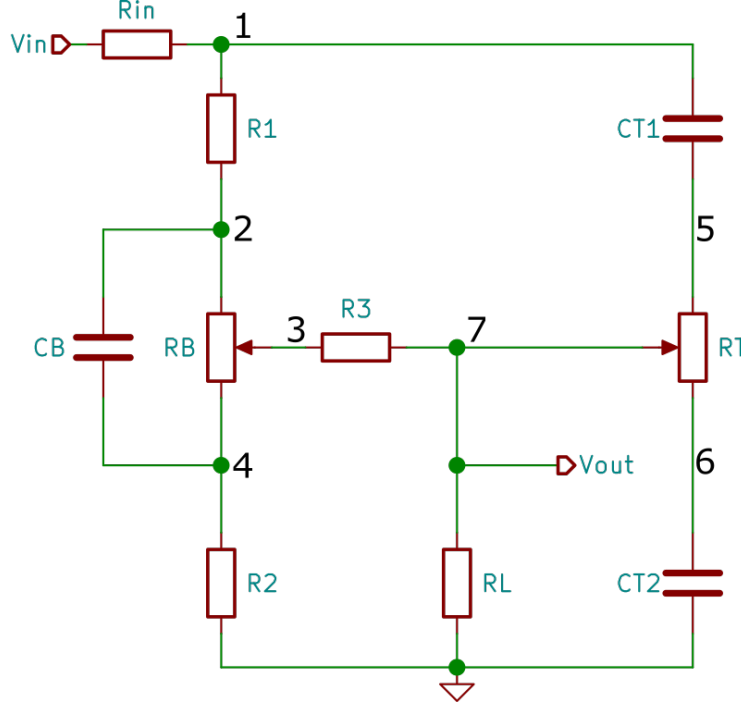


# Analysis of James & Baxandall Tone Control Circuits

Variation: Passive, single bass capacitor, dual treble capacitor

To find the frequency response of the circuit, the ratio  $\frac{V_{out}}{V_{in}}$  needs to be determined. Nodal analysis is performed to yield a system of linear equations, which are then placed in matrix form and solved using Cramer's rule.



## Nodal Analysis

**Node #1:** Using Kirchhoff's current law (KCL),

$$v_1\left(\frac{1}{R_{in}} + \frac{1}{R_1} + j\omega C_{T1}\right) + v_2\left(-\frac{1}{R_1}\right) + v_5(-j\omega C_{T1}) = \frac{V_{in}}{R_{in}}.$$

**Node #2:** Bass potentiometer  $R_B$  is modeled as two resistors,  $R_{B1}$  and  $R_{B2}$ , connected at the wiper.

$$v_1\left(-\frac{1}{R_1}\right) + v_2\left(\frac{1}{R_1} + \frac{1}{R_{B1}} + j\omega C_B\right) + v_3\left(-\frac{1}{R_{B1}}\right) + v_4(-j\omega C_B) = 0$$

**Node #3:**

$$v_2\left(-\frac{1}{R_{B1}}\right) + v_3\left(\frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{R_3}\right) + v_4\left(-\frac{1}{R_{B2}}\right) + V_{out}\left(-\frac{1}{R_3}\right) = 0$$

**Node #4:**

$$v_2(-j\omega C_B) + v_3\left(-\frac{1}{R_{B2}}\right) + v_4\left(\frac{1}{R_{B2}} + \frac{1}{R_2} + j\omega C_B\right) = 0$$

**Node #5:** Treble potentiometer  $R_T$  is modeled as resistors  $R_{T1}$  and  $R_{T2}$ , connected at the wiper.

$$v_1(-j\omega C_{T1}) + v_5(\frac{1}{R_{T1}} + j\omega C_{T1}) + V_{out}(-\frac{1}{R_{T1}}) = 0$$

**Node #6:**

$$v_6(\frac{1}{R_{T2}} + j\omega C_{T2}) + V_{out}(-\frac{1}{R_{T2}}) = 0$$

**Node #7:**

$$v_3(-\frac{1}{R_3}) + v_5(-\frac{1}{R_{T1}}) + v_6(-\frac{1}{R_{T2}}) + V_{out}(\frac{1}{R_3} + \frac{1}{R_{T1}} + \frac{1}{R_{T2}} + \frac{1}{R_L}) = 0$$

## Matrix Form

There are 7 node equations with 7 node voltage variables. These can be stated in matrix form

$$Ax = b$$

where  $A$  is a 7 x 7 matrix of the coefficients (admittances),  $x$  is a column vector of the variables (node voltages), and  $b$  is a column vector of the right-hand sides of the equations (inputs and constants).

$$\begin{bmatrix} \frac{1}{R_{in}} + \frac{1}{R_1} + j\omega C_{T1} & -\frac{1}{R_1} & 0 & 0 & -j\omega C_{T1} & 0 & 0 \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_{B1}} + j\omega C_B & -\frac{1}{R_{B1}} & -j\omega C_B & 0 & 0 & 0 \\ 0 & -\frac{1}{R_{B1}} & \frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{R_3} & -\frac{1}{R_{B2}} & 0 & 0 & -\frac{1}{R_3} \\ 0 & -j\omega C_B & -\frac{1}{R_{B2}} & \frac{1}{R_{B2}} + \frac{1}{R_2} + j\omega C_B & 0 & 0 & 0 \\ -j\omega C_{T1} & 0 & 0 & 0 & \frac{1}{R_{T1}} + j\omega C_{T1} & 0 & -\frac{1}{R_{T1}} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{R_{T2}} + j\omega C_{T2} & -\frac{1}{R_{T2}} \\ 0 & 0 & -\frac{1}{R_3} & 0 & -\frac{1}{R_{T1}} & -\frac{1}{R_{T2}} & \frac{1}{R_3} + \frac{1}{R_{T1}} + \frac{1}{R_{T2}} + \frac{1}{R_L} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ V_{out} \end{bmatrix} = \begin{bmatrix} \frac{V_{in}}{R_{in}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The output voltage,  $V_{out}$ , can now be found using Cramer's rule.

$$V_{out} = \frac{|A_7|}{|A|}$$

where matrix  $A_7$  is formed by replacing the 7th column of  $A$  with the contents of  $b$ . Since  $b$  contains only one non-zero element, the determinant of  $A_7$  is equal to that element multiplied by its cofactor.

$$|A_7| = \frac{V_{in}}{R_{in}} C_{1,7} = \frac{V_{in}}{R_{in}} (-1)^{1+7} (M_{1,7})$$

$$|A_7| = \frac{V_{in}}{R_{in}} M_{1,7}$$

where  $M_{1,7}$  is the determinant of  $A_7$  with row 1 and column 7 removed.

$$M_{1,7} = \begin{vmatrix} -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_{B1}} + j\omega C_B & -\frac{1}{R_{B1}} & -j\omega C_B & 0 & 0 \\ 0 & -\frac{1}{R_{B1}} & \frac{1}{R_{B1}} + \frac{1}{R_{B2}} + \frac{1}{R_3} & -\frac{1}{R_{B2}} & 0 & 0 \\ 0 & -j\omega C_B & -\frac{1}{R_{B2}} & \frac{1}{R_{B2}} + \frac{1}{R_2} + j\omega C_B & 0 & 0 \\ -j\omega C_{T1} & 0 & 0 & 0 & \frac{1}{R_{T1}} + j\omega C_{T1} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{R_{T2}} + j\omega C_{T2} \\ 0 & 0 & -\frac{1}{R_3} & 0 & -\frac{1}{R_{T1}} & -\frac{1}{R_{T2}} \end{vmatrix}$$

Substituting into the equation for  $V_{out}$ ,

$$V_{out} = \frac{V_{in}}{R_{in}} \frac{M_{1,7}}{|A|}.$$

The transfer function of the circuit is then found to be

$$\frac{V_{out}}{V_{in}} = \frac{1}{R_{in}} \frac{M_{1,7}}{|A|}.$$